The Optimization Test Environment

Martin Fuchs

in collaboration with Ferenc Domes, Hermann Schichl,
University of Vienna

June 18, 2009
Motivation

Testing is crucial for users who seek to

1. **compare solvers on suitable test sets**
   → solver benchmarks with large test sets

2. **compare solvers on single problems**
   → find a good solution of a specific problem

3. **adjust solver parameters**
   → improve the configuration of a particular solver

The Test Environment automatizes many of these (possibly little exciting) procedures for global optimization solvers.
Study background

Comparisons of global optimization solvers:

- one extensive study in 2005, based on the COCONUT Environment benchmark
- typically for black box problems only
- many interfaces of test environments are not very user friendly

! Generally, much effort has to be spent to conduct a solver comparison
Basic testing procedure

1. organize (possibly large) test libraries
2. solve selected problems with selected solvers
3. analyze, check and compare the results
4. create summary tables (e.g., in \LaTeX)
Notation for optimization problems

\[
\begin{align*}
\min \quad & f(x) \\
\text{s.t.} \quad & x \in \mathbf{x}, \\
& F(x) \in \mathbf{F}, \\
& x_i \in \mathbb{Z} \text{ for } i \in I.
\end{align*}
\] (1)

- \( \mathbf{x} = [\underline{x}, \bar{x}] = \{x \in \mathbb{R}^n \mid \underline{x} \leq x \leq \bar{x}\} \) is a box in \( \mathbb{R}^n \)
- \( f : \mathbf{x} \to \mathbb{R} \) is the objective function
- \( F : \mathbf{x} \to \mathbb{R}^m \) is a vector of constraint functions
- \( \mathbf{F} \) is a box in \( \mathbb{R}^m \)
- \( I \subseteq \{1, \ldots, n\} \) defines the integer components of \( x \)
Test problem examples

\[
\begin{align*}
\min_x \quad & x_2 \\
\text{s.t.} \quad & (x_1 - 0.5)^2 + x_2^2 = 1, \\
& (x_1 + 0.5)^2 + x_2^2 = 1, \\
& x_1 \in [-3, 3], x_2 \in [-3, 3].
\end{align*}
\]

\[\rightarrow \hat{x} = (0, -\sqrt{3}/2)^T \approx (0, -0.8660)^T\]
AMPL formulation

- var x1 >=-3, <=3;
- var x2 >=-3, <=3;

minimize obj:  x2;

s.t.  c1:  (x1-0.5)^2+x2^2=1;
s.t.  c2:  (x1+0.5)^2+x2^2=1;

our problem representations are DAGs (COCONUT Environment)

converters to DAGs will be available online for popular modeling languages (e.g., AMPL, GAMS)
Test problem examples ctd.

→ $x = (0, 0)^T$ is the only feasible point

→ no feasible solution
Interactive library management

demo
**Solver interface**

1. **solver selection**
2. **solver configuration** → setup paths and scripts
3. **test problem subset selection** → criteria editor
4. **solve command** → solve selected problems with selected solvers

→ [demo]
**Analyze**

- translate solver results to a unified format
  $\rightarrow \ .res$ files

<table>
<thead>
<tr>
<th>modelstatus</th>
<th>solver model status</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(i)</td>
<td>solver output for $\hat{x}_i$, $i = 1, \ldots, n$</td>
</tr>
<tr>
<td>obj</td>
<td>solver output for $f(\hat{x})$</td>
</tr>
<tr>
<td>infeas</td>
<td>feasibility distance provided by the solver</td>
</tr>
<tr>
<td>time</td>
<td>used CPU time to solve the problem</td>
</tr>
<tr>
<td>splits</td>
<td>number of splits made</td>
</tr>
</tbody>
</table>
Solution check

- let \( C := \{ x \in \mathbb{x} \mid x_I \in \mathbb{Z}, F(x) \in F \} \)

- compute a feasibility distance \( d_{\text{feas}} \)

- intuitively: \( \min_{y \in C} \| x - y \|_p \)

- computationally: componentwise feasibility

- account for scaling issues
Solution check ctd.

- parameters $\varepsilon (:= 10^{-6})$, $\kappa (:= 1)$, $\alpha (:= 10^{-6})$
- solver result $x_s, f_s$
- $x_s = x_s \pm \varepsilon \max(\|x_s\|_\infty, \kappa)$
- objective violation
  \[ v_o := \langle f(x_s) + f'(x_s)(x_s - x_s) - f_s \rangle \]
- box constraint violations
  \[ v_b := \langle x_s - x \rangle \]
- constraint violations
  \[ v_c := \langle F(x_s) + F'(x_s)(x_s - x_s) - F \rangle \]
- violation $v = \|(v_o, v_b, v_c)^T\|_p$ (we use $p = \infty$)
- if $v \leq \alpha \Rightarrow x_s$ feasible
Comparison of results

- \( J = \{x_1, \ldots, x_N\} \): solver results that passed the check

- **global solution** \( x_{\text{opt}} \):
  
  \( x_{\text{opt}} \in J \) and \( f(x_{\text{opt}}) \leq f(x_j) \) for all \( x_j \in J \)

- \( \tilde{x} \in J \), \( \tilde{x} \neq x_{\text{opt}} \) is also considered global if \( f(\tilde{x}) \) is sufficiently close to \( f(x_{\text{opt}}) \)

- otherwise \( \tilde{x} \) is a **local solution**
Comparison of results ctd.

- **best point:**

\[ x_{\text{best}} = \begin{cases} 
  x_{\text{opt}} & \text{if } J \neq \emptyset, \\
  \arg \min_{x \in J} d_{\text{feas}}(x) & \text{if } J = \emptyset.
\end{cases} \]

- if the best point was found by a default local solver → **easy location**

- otherwise → **hard location**
Performance assessment

Count for each solver

- the number of global solutions found

- the number of wrong and correct solver claims

Further performance measures for rigorous or black box solvers can be implemented in the future.
Summary tables

- automated creation of summary tables of the results in $\LaTeX$

→ just copy paste them to result sections of scientific publications

→ [demo]
Benchmark

COCONUT Environment benchmark:

- three libraries of global constrained optimization and constraint satisfaction problems
- over 1000 test problems in up to 20000 variables
- time limits depending on the problem size:

<table>
<thead>
<tr>
<th>size</th>
<th>(n)</th>
<th>timeout (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-9</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>10-99</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>100-999</td>
<td>1800</td>
</tr>
<tr>
<td>4</td>
<td>(\geq 1000)</td>
<td>.</td>
</tr>
</tbody>
</table>

We will collect solver benchmark results online.
## Preliminary results

<table>
<thead>
<tr>
<th></th>
<th>BARON summary statistics</th>
<th></th>
<th>KNITRO summary statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lib1</td>
<td>254</td>
<td>218</td>
<td>7</td>
</tr>
<tr>
<td>Lib2</td>
<td>599</td>
<td>532</td>
<td>21</td>
</tr>
<tr>
<td>Lib3</td>
<td>271</td>
<td>247</td>
<td>21</td>
</tr>
</tbody>
</table>

The Optimization Test Environment
Conclusions

Easy-to-use management of

- test libraries and problem subset selection
- solver configurations

Automatized

- solution check and comparison
- \texttt{\LaTeX} code creation
Final remarks

The preliminary benchmark results show

- global optimization is superior to local in most test cases
- BARON performs well as a global solver
- visit my website for updates

http://www.martin-fuchs.net