Uncertainty modeling with clouds in autonomous robust design optimization

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Two tasks:

1. autonomous design
   → design optimization algorithm

2. robust design
   → design safeguarded against uncertain perturbations
Overview

- Difficulties
- Uncertainty modeling by clouds
- Cloud generation
- Formulating the optimization problem
- Heuristics
- Application: Mars Exploration Rover
- Conclusions
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Introduction

Problems

- incomplete information, unformalized knowledge
- information updates
- high dimensionality of many real life problems
Goals

- apply to real life problems
- gather available uncertainty information
- create (or update) a corresponding cloud
- search for the optimal robust design
- iterate until satisfaction
Basic concept

Expert opinion

Uncertainty information

Cloud

Underlying model

Design objective

Optimization

Design point
Uncertainty elicitation

Uncertainty Elicitation

- Variable information:
  - Current variable: $A_{\text{max}}$
  - Unit:
  - Full variable name: maximal cross-sectional area

- A priori uncertainty information:
  - Normal distribution
  - Nominal value: 5.31
  - Parameters:
    - $\mu$: 5.31
    - $\sigma$: 0.063

Next
new theoretical approach: **clouds**

- in particular: potential based clouds
  - regions of worst-case relevant scenarios

- transform the uncertainty information into constraints for the optimization

- easily understandable

- computationally attractive
Potential clouds
Potential clouds

- $n$-dimensional random variable $\varepsilon$

- potential function $V : \mathbb{R}^n \to \mathbb{R}$

- lower $\alpha$-cut $C_\alpha := \{ \varepsilon \in \mathbb{R}^n \mid V(\varepsilon) \leq V_\alpha \}$
  contains at most a fraction of $\alpha$ of all possible scenarios

- upper $\alpha$-cut $\overline{C}_\alpha := \{ \varepsilon \in \mathbb{R}^n \mid V(\varepsilon) \leq \overline{V}_\alpha \}$
  contains at least a fraction of $\alpha$ of all possible scenarios

$\Rightarrow$ nested regions defining a potential cloud

- choice of $V$ should be computationally realizable, dictated by shape of a sample
Potential cloud generation

- generate a set of sample points $S$, similar to Latin Hypercube Sampling (LHS)
- weight the sample
  - weights chosen such that weighted marginal distributions match given marginal distributions
  - computed by solving a suitable LP
- choose a potential function $V$ (initially box shaped)
- compute weighted empirical distribution of $\{V(\varepsilon) \mid \varepsilon \in S\}$
- compute smoothed Kolmogorov-Smirnov (KS) bounds for the empirical distribution
KS bounds

\[ d_{KS} = \frac{\phi^{-1}(\alpha_{KS})}{\sqrt{N_S} + 0.12 + \frac{0.11}{\sqrt{N_S}}} \]

- \( \phi \) the Kolmogorov function
- \( \alpha_{KS} \) the confidence in the KS theorem
Potential cloud generation
Potential choice

Two natural, computationally tractable special cases

1. box shape

2. ellipsoid shape
Potential choice

\[ C_{0.95} \]
Polyhedron potential

- Polyhedral $\alpha$-cut $A(\varepsilon - m) \leq b$

- $V_p(\varepsilon) := \max_k \frac{(A(\varepsilon - m))_k}{b_k}$

- $A$ sparse
  - computationally less expensive
  - easy interpretation
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Uncertainty elicitation and modeling

Uncertainty elicitation

Scenario Exclusion

Projection:
- x-axis:
  - uncvr1
- y-axis:
  - uncvr2

Couple x and y axes

Add Constraint
Remove Constraint

Constraint Selection:
- uncvr1 vs uncvr2 ≠ 1
Basic concept

- Expert opinion
  - Uncertainty information
    - Cloud
      - Design objective
        - Optimization
          - Design point
      - Underlying model
Cloud constraints

- choose confidence level $\alpha$

- cloud given by the regions $C_\alpha$, $\bar{C}_\alpha$

- search for the worst-case scenario in $C_\alpha$ or $\bar{C}_\alpha$

- embed constraint in optimization problem formulation
Optimization problem

\[
\begin{align*}
\min_{\theta} \quad & \max_{x,z,\varepsilon} \quad g(x) \\
\text{s.t.} \quad & z = Z(\theta) + \varepsilon \\
& G(x, z) = 0 \\
& \theta \in T \\
& V(\varepsilon) \leq V_\alpha
\end{align*}
\]

(objective functions)

(table constraints)

(functional constraints)

(selection constraints)

(cloud constraint)
Notations

$$\min_{\theta} \max_{x,z,\varepsilon} g(x)$$ (objective functions)

- $\theta$ design point
- $x$ vector containing all output variables
- $z$ vector containing all input variables, consists of external inputs and design variables
- $\varepsilon$ random variable
- $g(x)$ design objective
Notations

\[
\text{s.t.} \quad z = Z(\theta) + \varepsilon \quad \text{(table constraints)}
\]

- assign to each \( \theta \) a vector \( z \) of input variables
- value of \( z \) is nominal entry from \( Z(\theta) \) plus its error \( \varepsilon \)
Notations

\[ G(x, z) = 0 \]  \hspace{1cm} \text{(functional constraints)}

- express the functional relationships defined in the underlying model
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Notations

\[ \theta \in T \]  
(selection constraints)

- \( T \) the set of all possible designs
Notations

\[ V(\varepsilon) \leq V_\alpha \]  
(cloud constraint)

- involves potential function level sets
- requires \( \varepsilon \) to be in lower \( \alpha \) cut
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Design optimization

Difficulties

- Mixed Integer Nonlinear Programming (MINLP)
  - potentially combinatorial explosion

- Bi-level problem
  - no derivatives in outer level
  - expensive objective function

- Black-box functional constraints
Heuristics

- **inner level**: solved by a linear program

- **outer level**: different strategies
  - **Snobfit**: fits a quadratic model of the objective function and minimizes this model
  - Evolutionary algorithm
  - Separable underestimation
Heuristics - outer level

⇒ starting points for a local search

⇒ enhance chance to find the global optimum
   (no guarantee)
Basic concept

- Expert opinion
  - Uncertainty information
    - Cloud
  - Underlying model
    - Optimization
      - Design objective
        - Design point
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Design optimization

Uncertainty elicitation
Application example

**Mars Exploration Rover mission (MER)**
- April 2000: official start of MER design
- June 2003: first rover launched
- July 2003: second rover launched
- November, December 2003: end of cruise stage

We focus on
- **Attitude Determination and Control System (ADCS)**
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Application example

MER spacecraft
The objective is to minimize the total mass needed for the ADCS subsystem.

The decision to make is the choice of the thrusters for the ADCS from a set of available thrusters.

Total mass consists of the total fuel and the mass of the thrusters.

The fuel is computed by a MATLAB model, which implies functional constraints for the optimization.
Variable structure

- fixed parameters (speed of light in vacuum, gravity constant on earth,...)
- uncertain external input variables (engine misalignment angles, spin rates,...)
- design variables (thrust $F$, specific impulse $I_{sp}$, mass $m_{thrust}$ of a thruster)
- result variables (total mass,...)
### Design table

<table>
<thead>
<tr>
<th>θ</th>
<th>Thruster</th>
<th>$F/N$</th>
<th>$I_{sp}/s$</th>
<th>$m_{thrust}/kg$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aerojet MR-111C</td>
<td>0.27</td>
<td>210.0</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>EADS CHT 0.5</td>
<td>0.50</td>
<td>227.3</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>MBB Erno CHT 0.5</td>
<td>0.75</td>
<td>227.0</td>
<td>190</td>
</tr>
<tr>
<td>4</td>
<td>TRW MRE 0.1</td>
<td>0.80</td>
<td>216.0</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>Kaiser-Marquardt KMHS Model 10</td>
<td>1.0</td>
<td>226.0</td>
<td>330</td>
</tr>
<tr>
<td>6</td>
<td>EADS CHT 1</td>
<td>1.1</td>
<td>223.0</td>
<td>290</td>
</tr>
<tr>
<td>7</td>
<td>MBB Erno CHT 2.0</td>
<td>2.0</td>
<td>227.0</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>EADS CHT 2</td>
<td>2.0</td>
<td>227.0</td>
<td>200</td>
</tr>
<tr>
<td>9</td>
<td>EADS S4</td>
<td>4.0</td>
<td>284.9</td>
<td>290</td>
</tr>
<tr>
<td>10</td>
<td>Kaiser-Marquardt KMHS Model 17</td>
<td>4.5</td>
<td>230.0</td>
<td>380</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Uncertainty specification

<table>
<thead>
<tr>
<th>Variable</th>
<th>Probability Distribution</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$N(1.3, 0.0013)$</td>
<td>$m$</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>$N(0, 0.5)$</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>$N(0, 0.5)$</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\omega_{spin_0}$</td>
<td>$N(12, 1.33)$</td>
<td>$rpm$</td>
</tr>
<tr>
<td>$\omega_{spin_1}$</td>
<td>$N(2, 0.0667)$</td>
<td>$rpm$</td>
</tr>
<tr>
<td>$\omega_{spin_2}$</td>
<td>$\Gamma(11, 0.25)$</td>
<td>$rpm$</td>
</tr>
<tr>
<td>$\omega_{spin_3}$</td>
<td>$L(2, 0.0667)$</td>
<td>$rpm$</td>
</tr>
<tr>
<td>$\psi_{slew_1}$</td>
<td>$N(5, 0.5)$</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\psi_{slew_2}$</td>
<td>$N(50.45, 5)$</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\psi_{slew_3}$</td>
<td>$N(5.13, 0.5)$</td>
<td>$\circ$</td>
</tr>
</tbody>
</table>
Optimization results

- Minimize the objective function: total mass $m_{tot}$

- Optimal design choice: $\theta = 9$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.24</td>
<td>8.08</td>
</tr>
</tbody>
</table>
Optimization results

- Variation of the uncertainty information:
- Optimal design choice: $\theta = 17$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.38</td>
<td>9.49</td>
</tr>
</tbody>
</table>

- originally: $\theta = 9$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.24</td>
<td>8.08</td>
</tr>
</tbody>
</table>

⇒ design point sensitive to variation of uncertainty information
Optimization results

- Optimization on the nominal values:
  - Optimal design choice: $\theta = 3$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>2.68</td>
<td>8.75</td>
</tr>
</tbody>
</table>

- originally: $\theta = 9$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.24</td>
<td>8.08</td>
</tr>
</tbody>
</table>

$\Rightarrow$ design point sensitive to accounting for uncertainty
Optimization results

- Worst-case search in $3 \sigma$ boxes:

- Optimal design choice: $\theta = 9$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.24</td>
<td>5.56</td>
</tr>
</tbody>
</table>

- originally: $\theta = 9$

<table>
<thead>
<tr>
<th></th>
<th>Nominal values</th>
<th>Worst case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{tot}$</td>
<td>3.24</td>
<td>8.08</td>
</tr>
</tbody>
</table>

$\Rightarrow$ more rigorous accounting for uncertainty produces significantly different worst-case estimations
Conclusions

Clouds

- capture and process incomplete and unformalized uncertainty information
- allow for a simple uncertainty elicitation and information updating
- were applied to higher dimensional real life problems

These slides are available on-line at: http://www.martin-fuchs.net