Uncertainty modeling with clouds in autonomous robust design optimization

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Introduction

Two tasks:

1. autonomous design
   $\longrightarrow$ design optimization algorithm
2. robust design
   $\longrightarrow$ design safeguarded against uncertain perturbations

Overview

- Difficulties
- Uncertainty modeling by clouds
- Cloud generation
- Formulating the optimization problem
- Heuristics
- Application: Mars Exploration Rover
- Conclusions

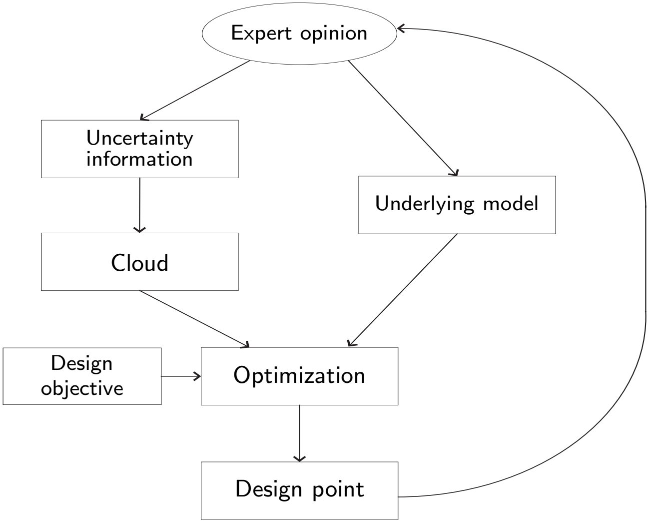
Problems

- incomplete information, unformalized knowledge
- information updates
- high dimensionality of many real life problems

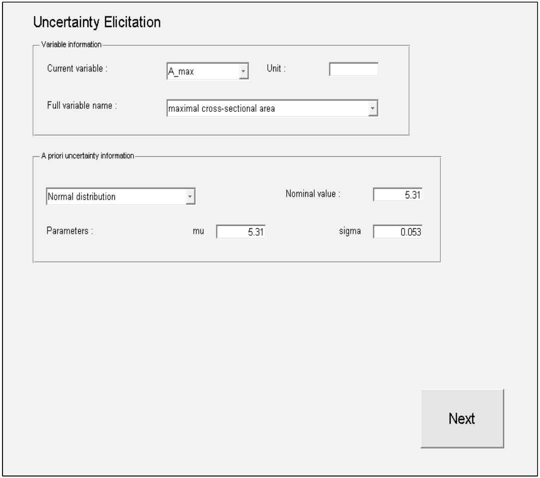
Goals

- apply to real life problems
- gather available uncertainty information
- create (or update) a corresponding **cloud**
- search for the optimal robust design
- iterate until satisfaction

Basic concept



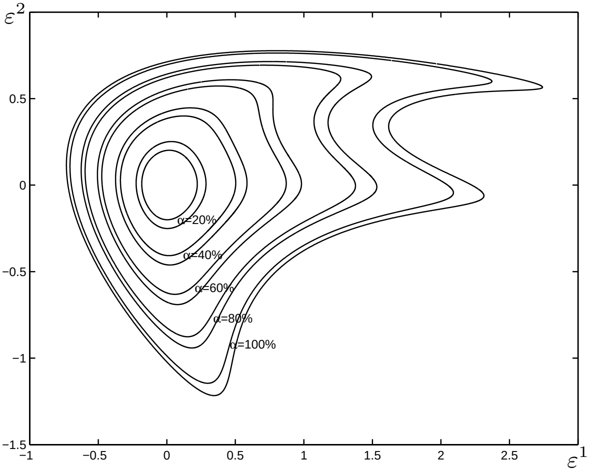
Uncertainty elicitation



Uncertainty modeling

- new theoretical approach: **clouds**
- in particular: potential based clouds
  $\Rightarrow$ regions of worst-case relevant scenarios
- transform the uncertainty information into constraints for the optimization
- easily understandable
- computationally attractive

Potential clouds



Potential clouds

- n-dimensional random variable ε
- potential function $V : \mathbb{R}^n \to \mathbb{R}$
- lower α-cut $\underline{C}_\alpha := \{\varepsilon \in \mathbb{R}^n \mid V(\varepsilon) \leq \underline{V}_\alpha\}$
  contains at most a fraction of α of all possible scenarios
- upper α-cut $\overline{C}_\alpha := \{\varepsilon \in \mathbb{R}^n \mid V(\varepsilon) \leq \overline{V}_\alpha\}$
  contains at least a fraction of α of all possible scenarios

$\Rightarrow$ nested regions defining a **potential cloud**

- choice of V should be computationally realizable,
  dictated by shape of a sample

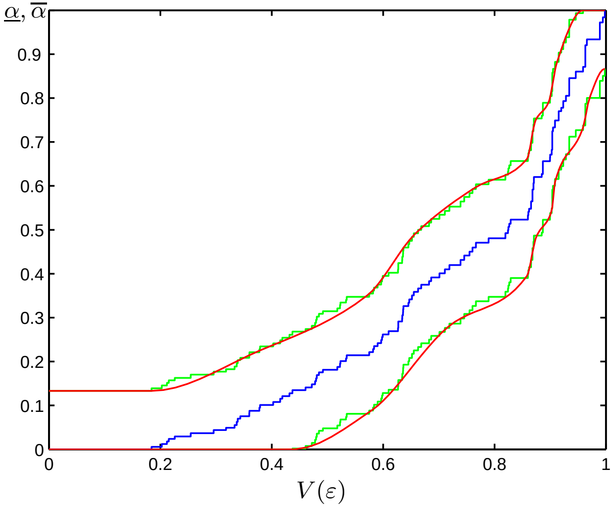
Potential cloud generation

- generate a set of sample points S,
  similar to Latin Hypercube Sampling (LHS)
- weight the sample
  - weights chosen such that weighted marginal distributions match given marginal distributions
  - computed by solving a suitable LP
- choose a potential function V (initially box shaped)
- compute weighted empirical distribution of $\{V(\varepsilon) \mid \varepsilon \in S\}$
- compute smoothed Kolmogorov-Smirnov (KS) bounds for the empirical distribution

KS bounds

- $d_{\mathrm{KS}} = \frac{\phi^{-1}(\alpha_{\mathrm{KS}})}{\sqrt{N_S}+0.12+\frac{0.11}{\sqrt{N_S}}}$
- ϕ the Kolmogorov function
- α_{KS} the confidence in the KS theorem

Potential cloud generation

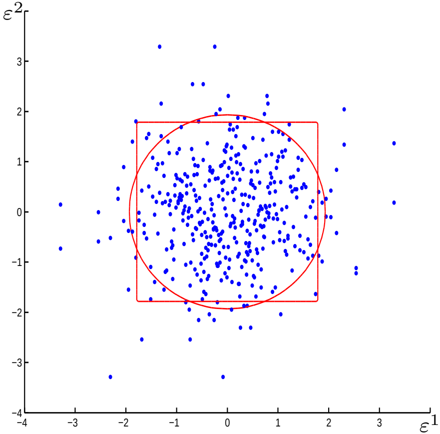


Potential choice

Two natural, computationally tractable special cases

1. box shape
2. ellipsoid shape

Potential choice

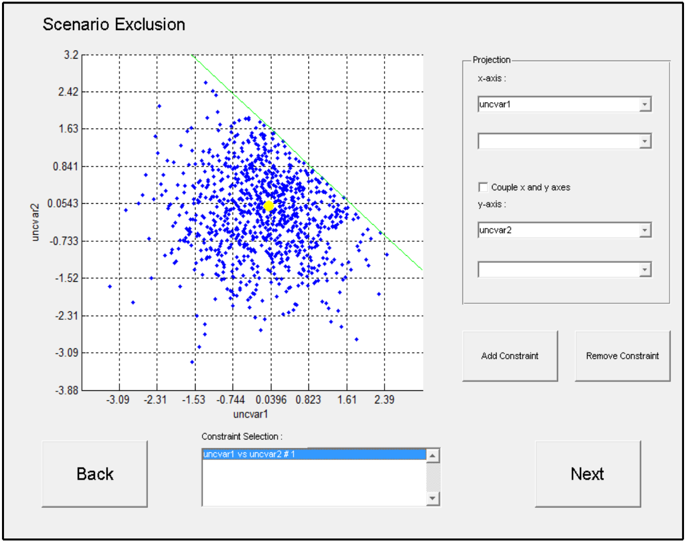


$\underline{C}_{0.95}$

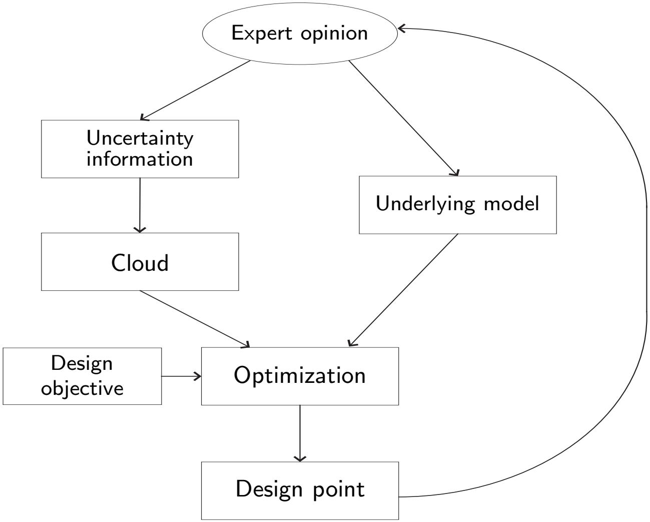
Polyhedron potential

- Polyhedral α-cut $A(\varepsilon - m) \leq b$
- $V_p(\varepsilon) := \max_k \frac{(A(\varepsilon - m))_k}{b_k}$
- A sparse
  - computationally less expensive
  - easy interpretation

Uncertainty elicitation



Basic concept



Cloud constraints

- choose confidence level α
- cloud given by the regions $\underline{C}_\alpha$, $\overline{C}_\alpha$
- search for the worst-case scenario in $\underline{C}_\alpha$ or $\overline{C}_\alpha$
- embed constraint in optimization problem formulation

Optimization problem

$$\min_{\theta} \max_{x,z,\varepsilon} g(x) \qquad \text{(objective functions)}$$

$$\text{s.t.} \quad z = Z(\theta) + \varepsilon \qquad \text{(table constraints)}$$

$$G(x, z) = 0 \qquad \text{(functional constraints)}$$

$$\theta \in T \qquad \text{(selection constraints)}$$

$$V(\varepsilon) \leq \underline{V}_{\alpha} \qquad \text{(cloud constraint)}$$

Notations

$$\min_{\theta} \max_{x,z,\varepsilon} g(x) \qquad \text{(objective functions)}$$

- θ design point
- x vector containing all output variables
- z vector containing all input variables, consists of external inputs and design variables
- ε random variable
- $g(x)$ design objective

Notations

$$\text{s.t.} \qquad z = Z(\theta) + \varepsilon \qquad \text{(table constraints)}$$

- assign to each θ a vector z of input variables
- value of z is nominal entry from $Z(\theta)$ plus its error ε

Notations

$$G(x, z) = 0 \qquad \text{(functional constraints)}$$

- express the functional relationships defined in the underlying model

Notations

$$\theta \in T \qquad \text{(selection constraints)}$$

- T the set of all possible designs

Notations

$$V(\varepsilon) \leq \underline{V}_\alpha \qquad \text{(cloud constraint)}$$

- involves potential function level sets
- requires ε to be in lower α cut

Difficulties

- Mixed Integer Nonlinear Programming (MINLP)
  - potentially combinatorial explosion
- Bi-level problem
  - no derivatives in outer level
  - expensive objective function
- Black-box functional constraints

Heuristics

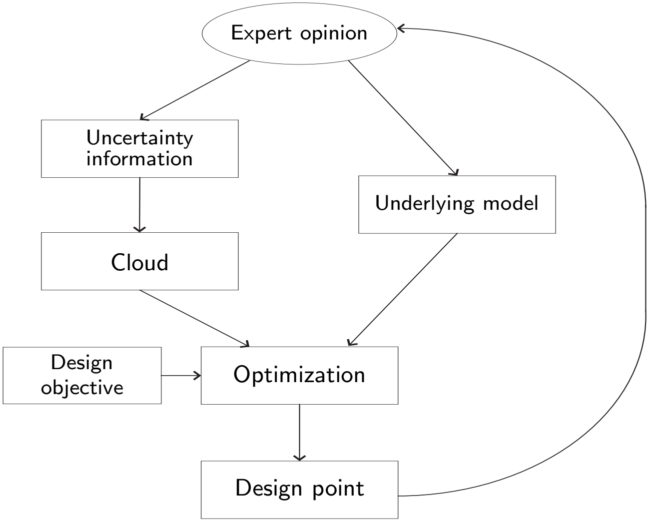
- inner level: solved by a linear program
- outer level: different strategies
  - SNOBFIT fits a quadratic model of the objective function and minimizes this model
  - Evolutionary algorithm
  - Separable underestimation

Heuristics - outer level

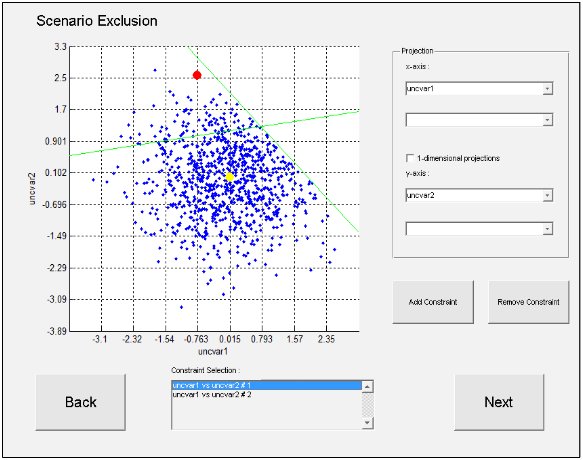
$\Rightarrow$ starting points for a local search

$\Rightarrow$ enhance chance to find the global optimum (no guarantee)

Basic concept



Uncertainty elicitation



Application example

Mars **E**xploration **R**over mission (**MER**)

- April 2000: official start of MER design
- June 2003: first rover launched
- July 2003: second rover launched
- November, December 2003: end of cruise stage

We focus on

- **A**ttitude **D**etermination and **C**ontrol **S**ystem (**ADCS**)

MER spacecraft

Problem specifications

- The objective is to minimize the total mass needed for the ADCS subsystem
- The decision to make is the choice of the thrusters for the ADCS from a set of available thrusters
- Total mass consists of the total fuel and the mass of the thrusters
- The fuel is computed by a MATLAB model
  $\Rightarrow$ functional constraints for the optimization

Variable structure

- fixed parameters (speed of light in vacuum, gravity constant on earth,...)
- uncertain external input variables (engine misalignment angles, spin rates,...)
- design variables (thrust F, specific impulse I_{sp}, mass m_{thrust} of a thruster)
- result variables (total mass,...)

Design table

θ	**Thruster**	F/N	I_{sp}/s	m_{thrust}/kg
1	Aerojet MR-111C	0.27	210.0	200
2	EADS CHT 0.5	0.50	227.3	200
3	MBB Erno CHT 0.5	0.75	227.0	190
4	TRW MRE 0.1	0.80	216.0	500
5	Kaiser-Marquardt KMHS Model 10	1.0	226.0	330
6	EADS CHT 1	1.1	223.0	290
7	MBB Erno CHT 2.0	2.0	227.0	200
8	EADS CHT 2	2.0	227.0	200
9	EADS S4	4.0	284.9	290
10	Kaiser-Marquardt KMHS Model 17	4.5	230.0	380
...				

Uncertainty specification

Variable	Probability Distribution	Unit
R	$N(1.3, 0.0013)$	m
δ_1	$N(0, 0.5)$	$\circ$
δ_2	$N(0, 0.5)$	$\circ$
ω_{spin0}	$N(12, 1.33)$	rpm
ω_{spin1}	$N(2, 0.0667)$	rpm
ω_{spin2}	$\Gamma(11, 0.25)$	rpm
ω_{spin3}	$L(2, 0.0667)$	rpm
ψ_{slew1}	$N(5, 0.5)$	$\circ$
ψ_{slew2}	$N(50.45, 5)$	$\circ$
ψ_{slew3}	$N(5.13, 0.5)$	$\circ$
...		

Optimization results

- Minimize the objective function: total mass m_{tot}
- Optimal design choice: $\theta = 9$

Variable	Nominal values	Worst case
m_{tot}	3.24	8.08

Optimization results

- Variation of the uncertainty information:
- Optimal design choice: $\theta = 17$

	Nominal values	Worst case
m_{tot}	3.38	9.49

- originally: $\theta = 9$

	Nominal values	Worst case
m_{tot}	3.24	8.08

$\Rightarrow$ design point sensitive to variation of uncertainty information

Optimization results

- Optimization on the nominal values:
- Optimal design choice: $\theta = 3$

	Nominal values	**Worst case**
m_{tot}	2.68	8.75

- originally: $\theta = 9$

	Nominal values	**Worst case**
m_{tot}	3.24	8.08

$\Rightarrow$ design point sensitive to accounting for uncertainty

Optimization results

- Worst-case search in 3 σ boxes:
- Optimal design choice: $\theta = 9$

	Nominal values	Worst case
m_{tot}	3.24	5.56

- originally: $\theta = 9$

	Nominal values	Worst case
m_{tot}	3.24	8.08

$\Rightarrow$ more rigorous accounting for uncertainty produces significantly different worst-case estimations

Conclusions

Clouds

- capture and process incomplete and unformalized uncertainty information
- allow for a simple uncertainty elicitation and information updating
- were applied to higher dimensional real life problems

These slides are available on-line at: `http://www.martin-fuchs.net`