Design Optimization Under Uncertainty Using Clouds

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Robust design optimization

1. Robustness
   - → safeguard against uncertain perturbations
   - → uncertainty modeling in higher dimensions

2. Design optimization
   - → computationally expensive black box objective functions
   - → possibly multiple objectives
     (e.g., cost, worst-case performance)

1. Uncertainty handling
2. Optimization
3. GUI
4. Conclusions

Difficulties in real-life applications I: Robustness issues

- Incomplete information
  - typical situation:
    intervals, marginal PDFs, but no correlation information
    insufficient amount of data, but information updates or
    unformalized information
- Aleatory and epistemic uncertainty
  - irreducible or reducible
- Curse of dimensionality
  - severe computational effort
- Avoid unjustified assumptions
  - e.g., PDF assumptions, independence assumption

Basic approach to account for robustness

- gather available uncertainty information
- create (or update) a corresponding **cloud**
- search for the optimal robust design
- iterate in case of information updates

Potential clouds

- n-dimensional random vector ε
- potential function $V : \mathbb{R}^n \to \mathbb{R}$

Construct

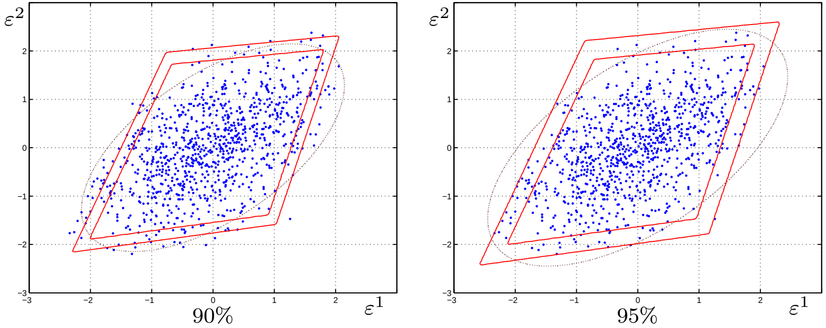
- lower α-cut $\underline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \underline{V}_\alpha\}$
  contains at most a fraction of α of all possible scenarios
- upper α-cut $\overline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \overline{V}_\alpha\}$
  contains at least a fraction of α of all possible scenarios

$\Rightarrow$ nested regions defining a **potential cloud**

How?

- regard $V(\varepsilon)$ as a 1-dimensional random variable
- find a rigorous enclosure of the CDF (p-box) of $V(\varepsilon)$

Example



- level sets of V chosen polyhedral shaped
- α-cuts reasonably approximate the confidence regions linearly

Worst-case search with clouds

$$\left.\begin{aligned} \max_{\varepsilon}\ & g(x) \\ \text{s.t.}\ & x = G(\theta, \varepsilon), \\ & \varepsilon \in \underline{C}_\alpha. \end{aligned}\right\} \qquad (1)$$

- θ, n_0-dimensional design point (fixed)
- $g : \mathbb{R}^m \to \mathbb{R}$, design objective (sought to be minimal)
- $G : \mathbb{R}^{n_0} \times \mathbb{R}^n \to \mathbb{R}^m$, functional constraints

Problem formulation

$$\left.\begin{array}{lll} \min\limits_{\theta} & \max\limits_{\varepsilon} & g(x) \\ \text{s.t.} & & G_0(x,z)=0, \\ & & z=Z(\theta)+\varepsilon, \\ & & \varepsilon \in \underline{C}_\alpha, \\ & & \theta \in \mathbf{T}. \end{array}\right\} \quad (2)$$

- $G_0 : \mathbb{R}^m \times \mathbb{R}^{n_z} \to \mathbb{R}$, functional constraints
- $\mathbf{T} \subseteq \mathbb{R}^{n_0}$, set of all possible designs
- $Z : \mathbb{R}^{n_0} \to \mathbb{R}^{n_z}$, design selection mapping

Difficulties in real-life applications II: Design optimization

- bilevel formulation,
  no derivatives in the outer level available
- mixed integer programming
- simulation based black box objective function
  $\longrightarrow$ computationally expensive evaluation
  $\longrightarrow$ strong nonlinearities, discontinuities
  $\longrightarrow$ hidden constraints

Design selection

- $\mathbf{T}$ is the set of all possible designs
- $\mathbf{T} = T^1 \times T^2 \times \cdots \times T^{n_0}$
- $T^i = \begin{cases} \{1, 2, \ldots, N_i\} & \text{in the discrete case,} \\ [\underline{\theta^i}, \overline{\theta^i}] & \text{in the continuous case.} \end{cases}$

Design selection example: discrete case

θ	**Thruster**	F/N	I_{sp}/s	m_{thrust}/kg
1	Aerojet MR-111C	0.27	210.0	200
2	EADS CHT 0.5	0.50	227.3	200
3	MBB Erno CHT 0.5	0.75	227.0	190
4	TRW MRE 0.1	0.80	216.0	500
5	Kaiser-Marquardt KMHS Model 10	1.0	226.0	330

- Typical $N_\tau \times n_\tau$ table τ with $N_\tau = 5$, $n_\tau = 3$, $n_0 = 1$, $\mathbf{T} = \{1, \dots, 5\}$
- contains specifications of design components and the associated choice variable θ
- τ imposes the table mapping $Z(\theta) = (\tau_{\theta,1}, \tau_{\theta,2}, \tau_{\theta,3})$, assigning an input parameter vector to a given design point θ
- G_0 is a physical model based on $Z(\mathbf{T})$, rather than on $\mathbf{T}$

Problem reformulation

- assume that the functional constraints $G_0(x,z)=0$ can be solved numerically for x given z, i.e., $x=G_{\mathrm{e}}(z)$
- substitute x and insert table constraints: $g(G_{\mathrm{e}}(Z(\theta)+\varepsilon)) =: G_{\mathrm{bb}}(\theta,\varepsilon)$, hence

$$
\begin{aligned}
\min_{\theta} \quad \max_{\varepsilon} \quad & G_{\mathrm{bb}}(\theta,\varepsilon) \\
\text{s.t.} \quad & \varepsilon \in \underline{C}_{\alpha}, \\
& \theta \in \mathbf{T}.
\end{aligned}
$$

Problem reformulation ctd.

- inner level: θ fixed

$$\begin{aligned} \max_{\varepsilon} \; & G_{\mathrm{bb}}(\theta, \varepsilon) \\ \text{s.t.} \; & \varepsilon \in \underline{C}_{\alpha} \end{aligned}$$

$\longrightarrow$ solved by a linear program

$\longrightarrow$ $\widehat{G}_{\mathrm{bb}}(\theta)$, solution of inner level

- outer level: black box optimization

$$\begin{aligned} \min_{\theta} \; & \widehat{G}_{\mathrm{bb}}(\theta) \\ \text{s.t.} \; & \theta \in \mathbf{T} \end{aligned}$$

$\longrightarrow$ different strategies

Heuristic approaches

- SNOBFIT (fits a quadratic model of the objective function and minimizes this model)
- Evolutionary algorithm
- Separable underestimation
- Splitting based on convex relaxation
- combine with local search

Separable underestimation: idea

- minimize a separable underestimator of $\widehat{G}_{\mathrm{bb}}(\theta)$:
  $q(\theta) := \sum_{i=1}^{n_0} q_i(\theta^i)$
- assume that we have function evaluations
  $\widehat{G}_{\mathrm{bb}_1}, \widehat{G}_{\mathrm{bb}_2}, \ldots, \widehat{G}_{\mathrm{bb}_{N_0}}$ for $\theta_1, \ldots, \theta_{N_0} \in \mathbf{T}$.
- define

$$q_i(\theta_l^i) := \begin{cases} q_{i,\theta_l^i} & \text{if } \theta^i \text{ discrete choice,} \\ q_{i1}\theta_l^i + q_{i2}{\theta_l^i}^2 & \text{if } \theta^i \text{ continuous choice.} \end{cases}$$

- formulate LP with constraints

$$\sum_{i=1}^{n_0} q_i(\theta_l^i) \le \widehat{G}_{\mathrm{bb}_l},\ l = 1, 2, \ldots, N_0.$$

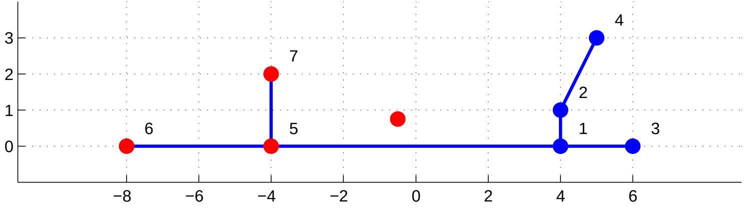
Convex relaxation based splitting: idea

- remember: G_0 (thus also $\widehat{G}_{\mathrm{bb}}$) is based on $Z(\mathbf{T})$, rather than on $\mathbf{T}$, so minimize $G_z(z) := \widehat{G}_{\mathrm{bb}}(Z^{-1}(z))$, s.t. $z \in Z(\mathbf{T})$
- convex relaxation of $Z(\mathbf{T})$:

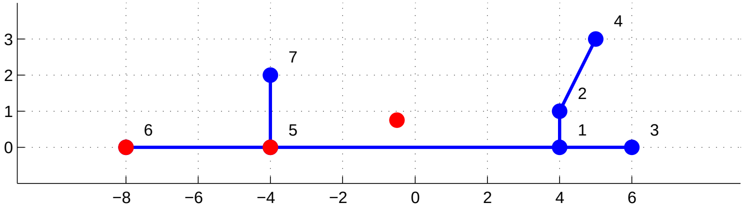
$$
\begin{aligned}
\min_{z,v,\lambda}\ & G_z(z) \\
\text{s.t.}\ & z = (v^1, \ldots, v^{n_0}), \\
& v^i = \sum_{j=1}^{N_i} \lambda_j^i Z^i(j) \text{ for discrete } T^i, \\
& \sum_{j=1}^{N_i} \lambda_j^i = 1 \text{ for discrete } T^i, \\
& \lambda_j^i \geq 0 \text{ for discrete } T^i, 1 \leq j \leq N_i, \\
& v^i \in [\underline{\theta^i}, \overline{\theta^i}] \text{ for continuous } T^i.
\end{aligned}
$$

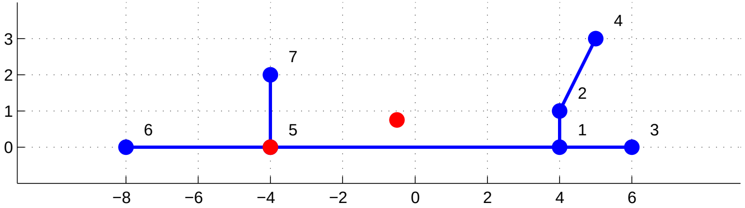
Convex relaxation based splitting ctd.

- use the coefficients of the convex relaxation as weights on the minimum spanning tree (MST) of $Z(\mathbf{T})$
- split the MST in two of parts of similar total weight
- Example:

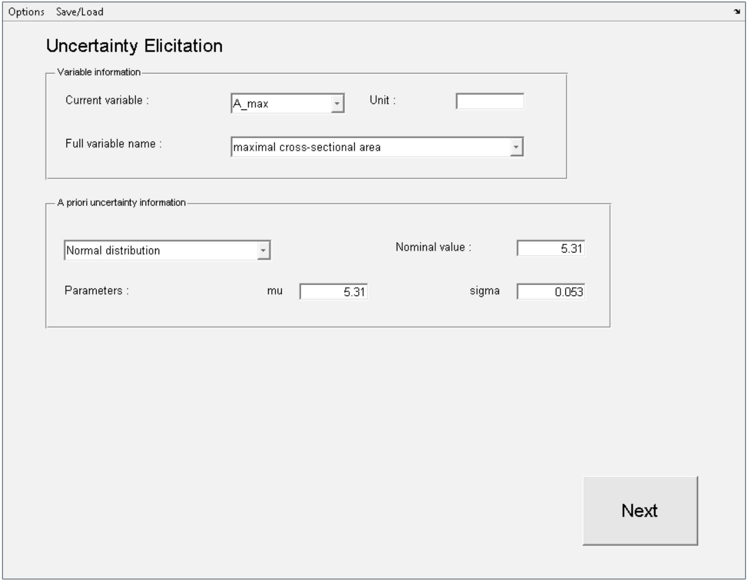


Convex relaxation based splitting ctd.

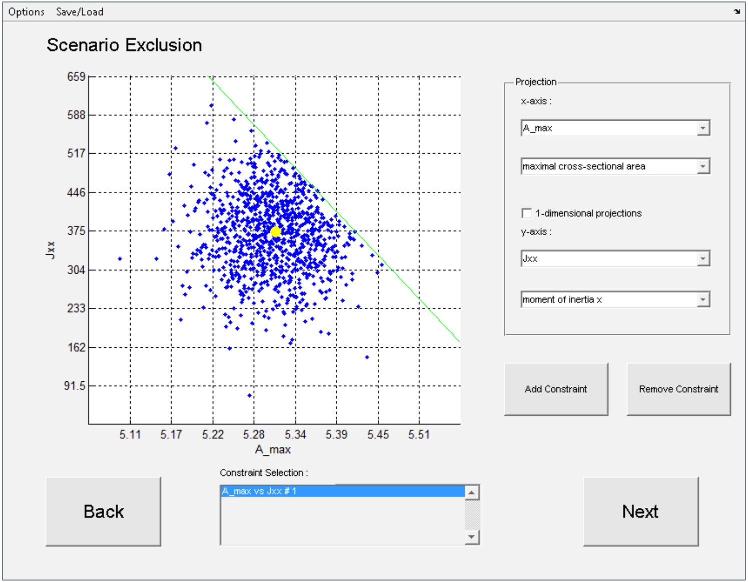




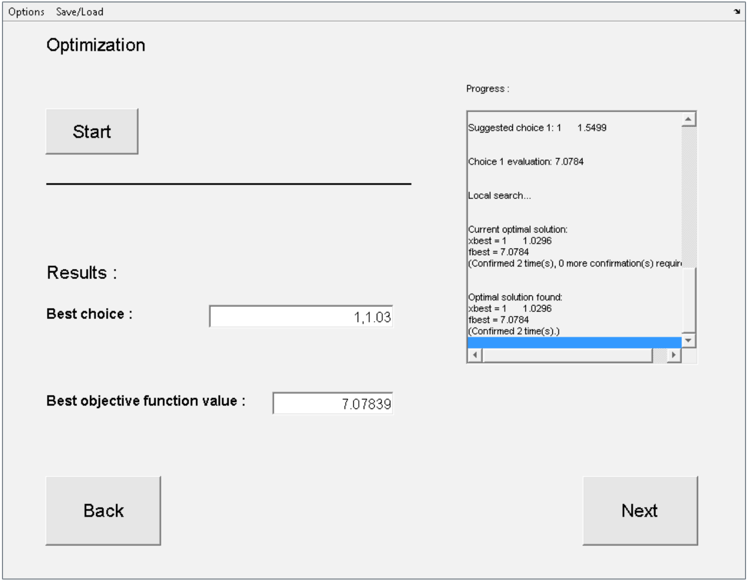
Graphical user interface (GUI)



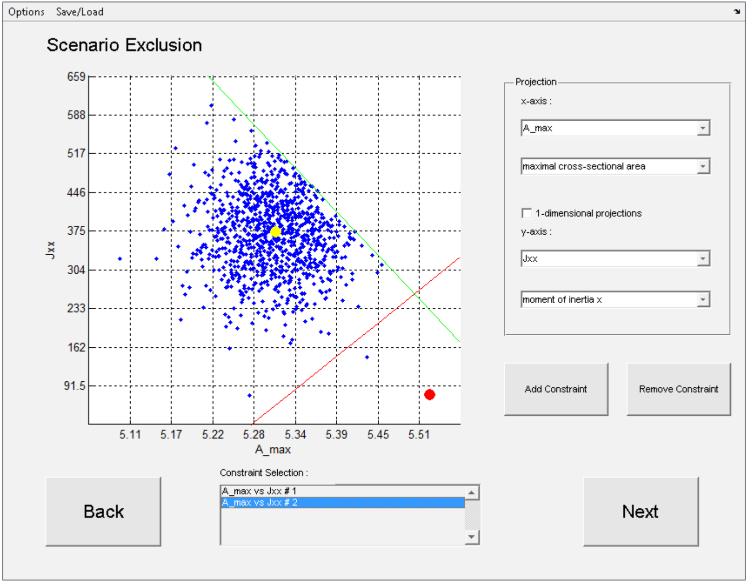
Uncertainty elicitation



Optimization phase



Adaptive uncertainty elicitation



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Conclusions

Cloud based design optimization (CBDO)

- captures and processes incomplete and unformalized information
- allows for an intuitive uncertainty elicitation and information updating
- solves successfully higher dimensional real-life design optimization problems

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