Cloud based design optimization

Martin Fuchs$^1$

$^1$CERFACS, Toulouse, France

July 21, 2009
Cloud based design optimization (CBDO) seeks to

- capture and model high-dimensional uncertainty information
  → by means of **clouds**

- improve robustness and optimality in real-life engineering design
  → bilevel mixed integer programming
Outline

1. Robust design optimization
2. Uncertainty modeling with clouds
3. Relations to other uncertainty models
4. Graphical user interface
Example discrete design choice

<table>
<thead>
<tr>
<th>θ</th>
<th>Thruster</th>
<th>$F/N$</th>
<th>$I_{sp}/s$</th>
<th>$m/kg$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aerojet MR-111C</td>
<td>0.27</td>
<td>210.0</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>EADS CHT 0.5</td>
<td>0.50</td>
<td>227.3</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>MBB Erno CHT 0.5</td>
<td>0.75</td>
<td>227.0</td>
<td>190</td>
</tr>
<tr>
<td>4</td>
<td>TRW MRE 0.1</td>
<td>0.80</td>
<td>216.0</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>Kaiser-Marquardt KMHS Model 10</td>
<td>1.0</td>
<td>226.0</td>
<td>330</td>
</tr>
</tbody>
</table>

- typical $N_τ \times n_τ$ table $τ$ with $N_τ = 5$, $n_τ = 3$
- contains specifications of design components and the associated choice variable $θ$
- table mapping $Z(θ) = (τ_θ,1, τ_θ,2, τ_θ,3)$, $z_0 = Z(θ)$ assigns a nominal input parameter vector $z_0$ to a given design point $θ$
Notation

- $\theta$, $n_0$-dimensional design point
- $T \subseteq \mathbb{R}^{n_0}$, set of all possible designs
- $\varepsilon$, $n$-dimensional random vector
- $g : \mathbb{R}^m \rightarrow \mathbb{R}$, design objective
- $G : \mathbb{R}^{n_0} \times \mathbb{R}^n \rightarrow \mathbb{R}^m$, functional constraints
Problem formulation

\[
\begin{align*}
\min_{\theta} \max_{\varepsilon} & \quad g(x) \\
\text{s.t.} & \quad x = G(\theta, \varepsilon), \\
& \quad \theta \in T, \\
& \quad \varepsilon \in \mathcal{C}.
\end{align*}
\]

(1)

- bilevel mixed integer programming
- nonlinear or black box objective function
- $\varepsilon \in \mathcal{C}$ represents the uncertainties
Potential clouds

- $n$-dimensional random vector $\varepsilon$
- potential function $V : \mathbb{R}^n \to \mathbb{R}$

Construct

- lower $\alpha$-cut $C_{\alpha} := \{ x \in \mathbb{R}^n | V(x) \leq V_\alpha \}$ contains at most a fraction of $\alpha$ of all possible scenarios
- upper $\alpha$-cut $\overline{C}_{\alpha} := \{ x \in \mathbb{R}^n | V(x) \leq \overline{V}_\alpha \}$ contains at least a fraction of $\alpha$ of all possible scenarios

$\Rightarrow$ nested regions defining a potential cloud
Difficulties to be tackled

- Incomplete information
  - scarce data, conflicting, or unformalized information
  - typically available:
    - intervals, marginal CDFs, information updates
  - typically **not** available:
    - correlation information, sufficient amount of data

- High dimensionality of many real-life problems

- Avoid unjustified assumptions
Example

- generate data from an $\mathcal{N}(0, \Sigma)$ distribution with
  \[
  \Sigma = \begin{pmatrix}
  1 & 0.6 \\
  0.6 & 1
  \end{pmatrix}
  \]

- assume that an expert only knows the data, but not the probability distribution

- the expert may have knowledge about the dependence of the variables

- polyhedral constraints model this knowledge
Example ctd.

- $\alpha$-cuts reasonably approximate the confidence regions linearly.
\textit{p-boxes}

- find rigorous enclosure of the CDF of a univariate random variable

- computational difficulties in higher dimensions

Relationship to potential clouds:

- regard $V(\varepsilon)$ as a 1-dimensional random variable

- a \textit{p-box} for $V(\varepsilon)$ can be considered as a cloud
Dempster-Shafer structures

- combines expert opinions modeled by finitely many sets $\mathcal{A}_k$ of focal sets together with a basic probability assignment $m_k$

  $\rightarrow$ lower and upper fuzzy measures $Bel, Pl$

- computationally expensive in higher dimensions

Relationship to potential clouds:

- $A_i := \overline{C_{\alpha_i}} \setminus C_{\alpha_i}$,
  
  $m(A_1) = \alpha_1, m(A_i) = \alpha_i - \alpha_{i-1}, i = 2, \ldots, N$,

  constructs a DS-structure from a cloud

- a DS-structure on $X := V(\varepsilon)$ corresponds to a cloud using $Bel(\{X \leq t\}), Pl(\{X \leq t\})$
Fuzzy sets and \( \alpha \)-level optimization

- membership function \( \mu \) for \( \varepsilon \), \( \alpha \)-cut \( C_\alpha \)
- seek: membership function \( \mu_f \) of a function \( f(\varepsilon) \), \( f : \mathbb{R}^n \rightarrow \mathbb{R} \)

\[ C_{f\alpha_i} = [f_{i*}, f_i^*], \text{ where } f_{i*}, f_i^* \text{ are solutions of} \]

\[ \min_{\varepsilon \in C_{\alpha_i}} f(\varepsilon), \text{ and} \]

\[ \max_{\varepsilon \in C_{\alpha_i}} f(\varepsilon), \text{ for different } \alpha_i \]

\[ \mu_f(x) = \sup_\alpha \min(\alpha, 1_{C_{f\alpha}}(x)) \]

- in \( n \) dimensions one assumes non-interactivity,
  \( C_{\alpha_i} \) is typically a hypercube
Fuzzy sets and $\alpha$-level optimization ctd.

Relationship to potential clouds:

- consider $C_\alpha$, $\overline{C}_\alpha$ as $\alpha$-cuts of a multidimensional interval valued membership function
- the direction fuzzy set $\rightarrow$ cloud requires consistent possibility/necessity measures
- $\alpha$-level optimization could be used to construct $C_{f\alpha}$, $\overline{C}_{f\alpha}$, i.e., functions of clouds
- expert knowledge about interactivity could be modeled for fuzzy sets similar to clouds
### Graphical user interface (GUI)

#### Uncertainty Elicitation

**Variable information**

Current variable: A\(_{\text{max}}\)

Unit: 

Full variable name: maximal cross-sectional area

**A priori uncertainty information**

Normal distribution: 

Nominal value: 5.31

Parameters: 

\[ \begin{align*}
\mu & = 5.31 \\
\sigma & = 0.053
\end{align*} \]
Uncertainty elicitation
Optimization phase

Optimization

Start

Results:

Best choice:

Best objective function value:

7.07839
Adaptive uncertainty elicitation
Conclusions

Cloud based design optimization (CBDO)

- models incomplete and unformalized information towards robust optimization
- allows for a simple uncertainty elicitation and information updating

These slides are available on-line at:  http://www.martin-fuchs.net