Cloud based design optimization

Martin Fuchs[1]

[1]CERFACS, Toulouse, France

July 21, 2009

Motivation

Cloud based design optimization (CBDO) seeks to

- capture and model high-dimensional uncertainty information
  - → by means of **clouds**
- improve robustness and optimality in real-life engineering design
  - → bilevel mixed integer programming

Outline

1. Robust design optimization
2. Uncertainty modeling with clouds
3. Relations to other uncertainty models
4. Graphical user interface

Example discrete design choice

θ	**Thruster**	F/N	I_{sp}/s	m/kg
1	Aerojet MR-111C	0.27	210.0	200
2	EADS CHT 0.5	0.50	227.3	200
3	MBB Erno CHT 0.5	0.75	227.0	190
4	TRW MRE 0.1	0.80	216.0	500
5	Kaiser-Marquardt KMHS Model 10	1.0	226.0	330

- typical $N_\tau \times n_\tau$ table τ with $N_\tau = 5$, $n_\tau = 3$
- contains specifications of design components and the associated choice variable θ

→ table mapping $Z(\theta) = (\tau_{\theta,1}, \tau_{\theta,2}, \tau_{\theta,3})$, $z_0 = Z(\theta)$ assigns a nominal input parameter vector z_0 to a given design point θ

Notation

- θ, n_0-dimensional design point
- $\mathbf{T} \subseteq \mathbb{R}^{n_0}$, set of all possible designs
- ε, n-dimensional random vector
- $g : \mathbb{R}^m \longrightarrow \mathbb{R}$, design objective
- $G : \mathbb{R}^{n_0} \times \mathbb{R}^n \longrightarrow \mathbb{R}^m$, functional constraints

Problem formulation

$$\left.\begin{array}{lll} \min\limits_{\theta} & \max\limits_{\varepsilon} & g(x) \\ \text{s.t.} & & x = G(\theta, \varepsilon), \\ & & \theta \in \mathbf{T}, \\ & & \varepsilon \in \mathcal{C}. \end{array}\right\} \quad (1)$$

- bilevel mixed integer programming
- nonlinear or black box objective function
- $\varepsilon \in \mathcal{C}$ represents the uncertainties

Potential clouds

- n-dimensional random vector ε
- potential function $V : \mathbb{R}^n \to \mathbb{R}$

Construct

- lower α-cut $\underline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \underline{V}_\alpha\}$
  contains at most a fraction of α of all possible scenarios
- upper α-cut $\overline{C}_\alpha := \{x \in \mathbb{R}^n \mid V(x) \leq \overline{V}_\alpha\}$
  contains at least a fraction of α of all possible scenarios

$\Rightarrow$ nested regions defining a **potential cloud**

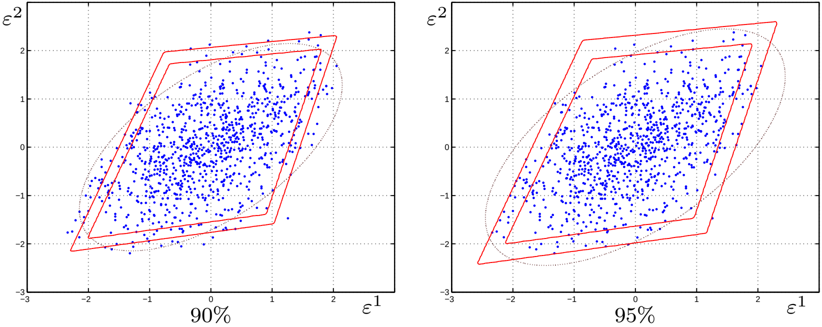
Difficulties to be tackled

- Incomplete information
    - scarce data, conflicting, or unformalized information
    - typically available:
      intervals, marginal CDFs, information updates
    - typically **not** available:
      correlation information, sufficient amount of data
- High dimensionality of many real-life problems
- Avoid unjustified assumptions

Example

- generate data from an $N(0,\Sigma)$ distribution with $\Sigma = \begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix}$
- assume that an expert only knows the data, but not the probability distribution
- the expert may have knowledge about the dependence of the variables
- polyhedral constraints model this knowledge

Example ctd.



90% 95%

- α-cuts reasonably approximate the confidence regions linearly

p-boxes

- find rigorous enclosure of the CDF of a univariate random variable
- computational difficulties in higher dimensions

Relationship to potential clouds:

- regard $V(\varepsilon)$ as a 1-dimensional random variable
- a p-box for $V(\varepsilon)$ can be considered as a cloud

Dempster-Shafer structures

- combines expert opinions modeled by finitely many sets $\mathcal{A}_k$ of focal sets together with a basic probability assignment m_k

$\rightarrow$ lower and upper fuzzy measures Bel, Pl

- computationally expensive in higher dimensions

Relationship to potential clouds:

- $A_i := \overline{C}_{\alpha_i} \backslash \underline{C}_{\alpha_i}$,
  $m(A_1) = \alpha_1, m(A_i) = \alpha_i - \alpha_{i-1}, i = 2, \ldots, N$,
  constructs a DS-structure from a cloud
- a DS-structure on $X := V(\varepsilon)$ corresponds to a cloud using $Bel(\{X \leq t\}), Pl(\{X \leq t\})$

Fuzzy sets and α-level optimization

- membership function μ for ε, α-cut C_α
- seek: membership function μ_f of a function $f(\varepsilon)$, $f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$\longrightarrow$ compute α-cuts belonging to μ_f by $C_{f_{\alpha_i}} = [f_{i_*}, f_i^*]$, where f_{i_*}, f_i^* are solutions of

$$\min_{\varepsilon \in C_{\alpha_i}} f(\varepsilon), \text{ and}$$

$$\max_{\varepsilon \in C_{\alpha_i}} f(\varepsilon), \text{ for different } \alpha_i$$

$\longrightarrow$ $\mu_f(x) = \sup_\alpha \min(\alpha, 1_{C_{f_\alpha}}(x))$

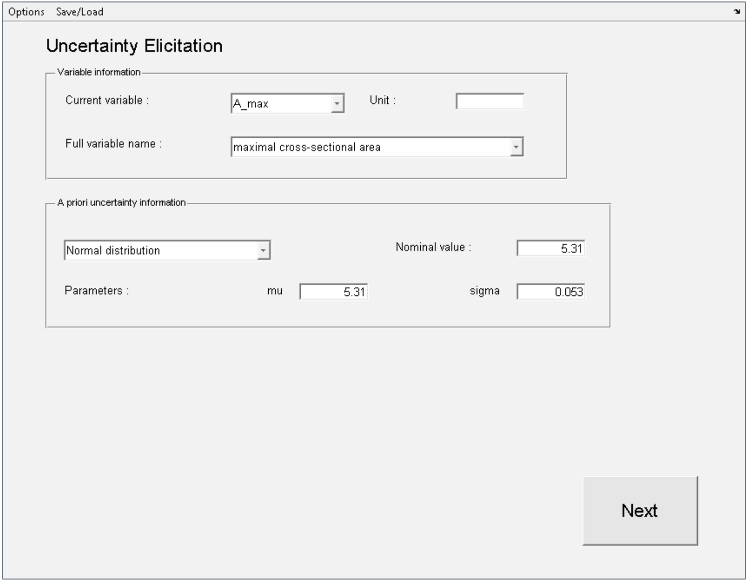
- in n dimensions one assumes non-interactivity, C_{α_i} is typically a hypercube

Fuzzy sets and α-level optimization ctd.

Relationship to potential clouds:

- consider $\underline{C}_\alpha$, $\overline{C}_\alpha$ as α-cuts of a multidimensional interval valued membership function
- the direction fuzzy set → cloud requires consistent possibility/necessity measures
- α-level optimization could be used to construct $\underline{C}_{f\,\alpha}$, $\overline{C}_{f\,\alpha}$, i.e., functions of clouds
- expert knowledge about interactivity could be modeled for fuzzy sets similar to clouds

Graphical user interface (GUI)



Uncertainty Elicitation

Variable information

Current variable : A_max

Unit :

Full variable name : maximal cross-sectional area

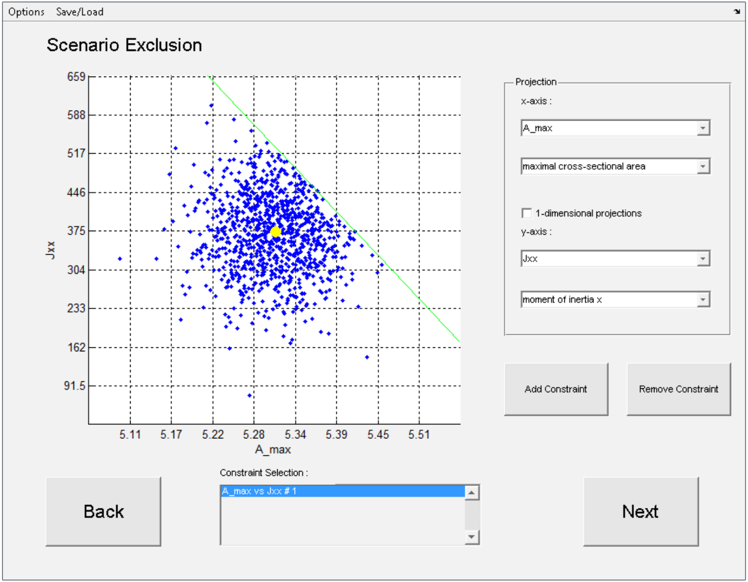
A priori uncertainty information

Normal distribution

Nominal value : 5.31

Parameters : mu 5.31 sigma 0.053

Uncertainty elicitation



x-axis : A_max

maximal cross-sectional area

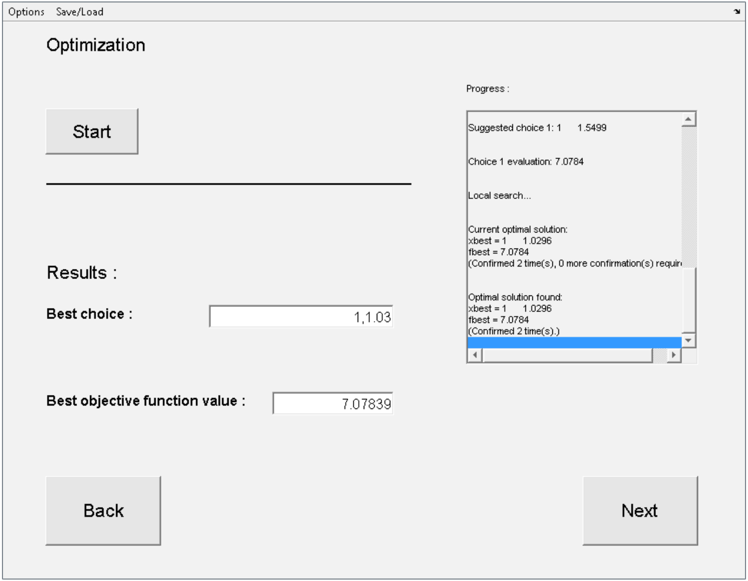
☐ 1-dimensional projections

y-axis : Jxx

moment of inertia x

Constraint Selection : A_max vs Jxx # 1

Optimization phase



Optimization

Progress :

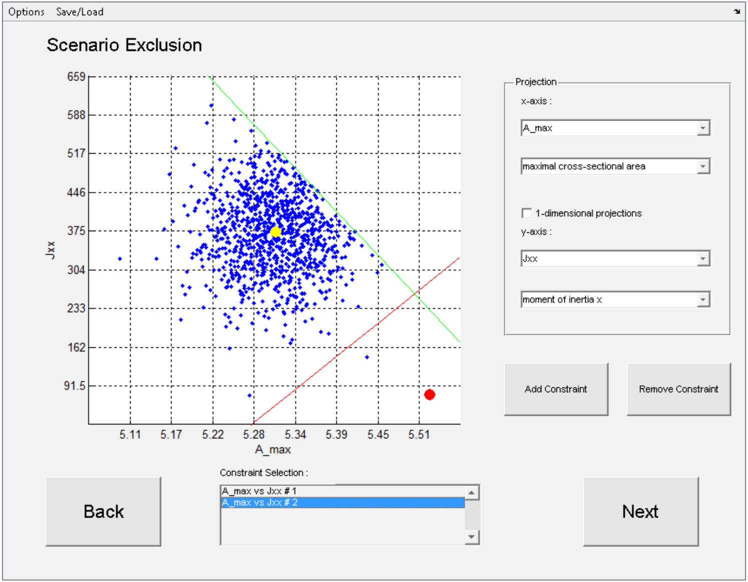
```
Suggested choice 1: 1     1.5499

Choice 1 evaluation: 7.0784

Local search...

Current optimal solution:
xbest = 1     1.0296
fbest = 7.0784
(Confirmed 2 time(s), 0 more confirmation(s) requir

Optimal solution found:
xbest = 1     1.0296
fbest = 7.0784
(Confirmed 2 time(s).)
```

Results :

Best choice : 1,1.03

Best objective function value : 7.07839

Adaptive uncertainty elicitation



x-axis : A_max

maximal cross-sectional area

☐ 1-dimensional projections

y-axis : Jxx

moment of inertia x

Constraint Selection :

A_max vs Jxx # 1

A_max vs Jxx # 2

Conclusions

Cloud based design optimization (CBDO)

- models incomplete and unformalized information towards robust optimization
- allows for a simple uncertainty elicitation and information updating

These slides are available on-line at: `http://www.martin-fuchs.net`