Uncertainty modeling in autonomous robust spacecraft system design

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Introduction

Two tasks:

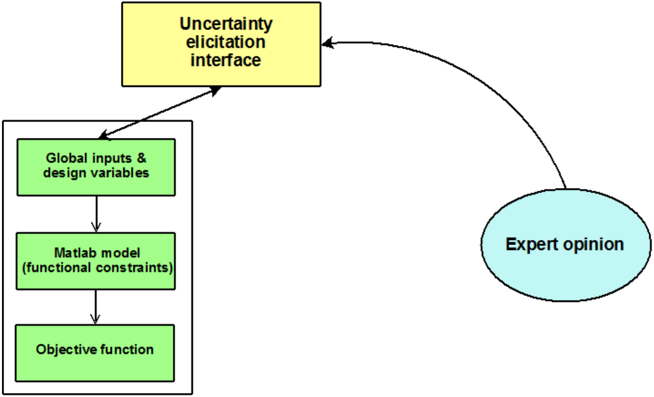
1. autonomous design $\longrightarrow$ design optimization algorithm
2. robust design $\longrightarrow$ design safeguarded against uncertain perturbations

Introduction

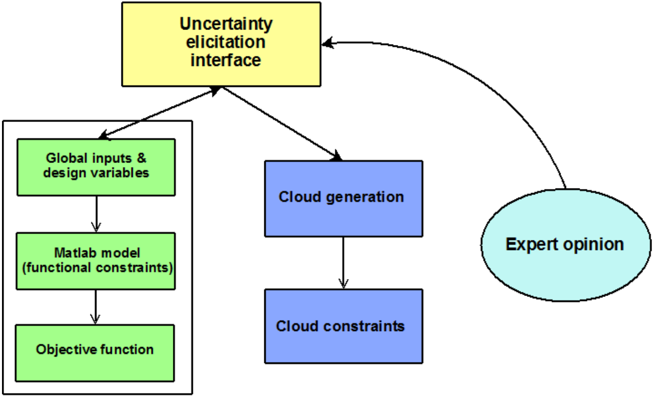
Development of a sophisticated tool for uncertainty handling:

- gathers all available uncertainty information
- models the information with the new theory of clouds
- processes the information to perform a worst-case analysis of a given design
- searches for the design with the best worst-case, i.e. the optimal robust design
- applied to problems in spacecraft system design

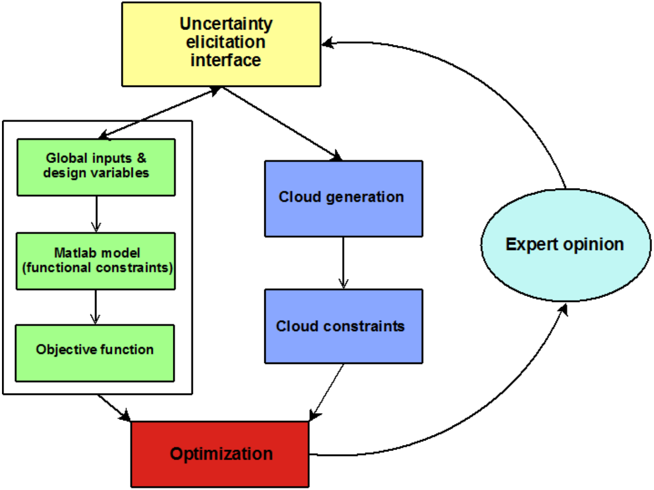
Basic concept



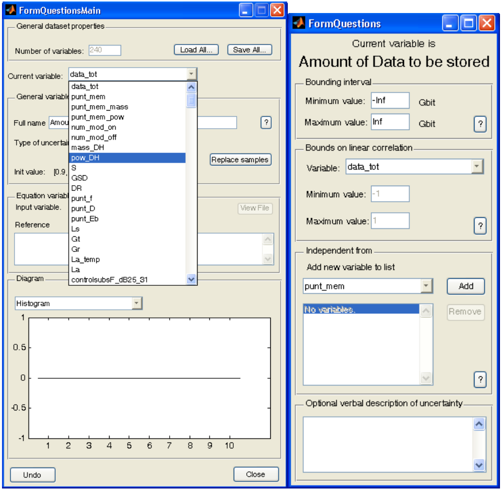
Basic concept



Basic concept



Uncertainty elicitation



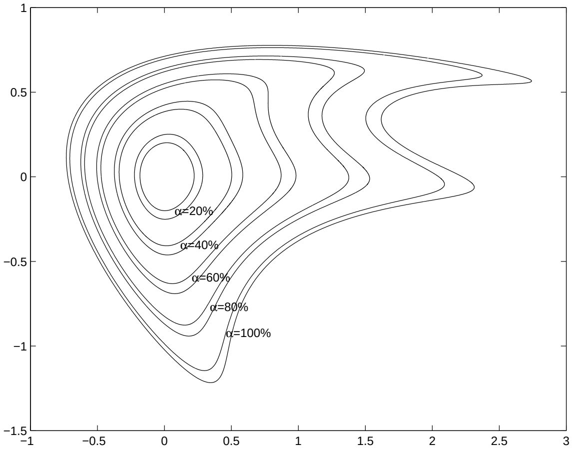
Uncertainty modeling

Second step:

- process the uncertainty information to constraints for the optimization
- new theoretical approach: clouds
- in particular: potential based clouds

⇒ regions of relevant scenarios affecting the worst-case for a given confidence level

Uncertainty modeling



Cloud generation

Assume the uncertainty information is given by marginal distributions or boxes for the vector ε of uncertain variables:

- generate a set of sample points S
- weight the sample
- choose a potential function V (initially box shaped)
- compute the empirical distribution for $\{V(\varepsilon), \varepsilon \in S\}$
- compute a tube around it that considers the sample size and the dimensionality

Cloud generation

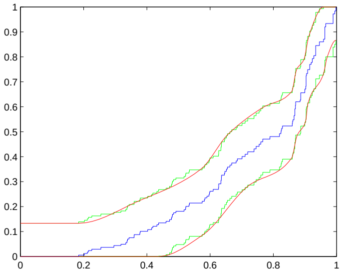


Figure: Tube, smooth lower bounds $\underline{\alpha}(V(\varepsilon))$ and upper bounds $\overline{\alpha}(V(\varepsilon))$, the mapping $\varepsilon \longrightarrow [\underline{\alpha}(V(\varepsilon)), \overline{\alpha}(V(\varepsilon))]$ which is a closed interval in $\mathbb{R}$ is a potential based cloud.

Cloud generation

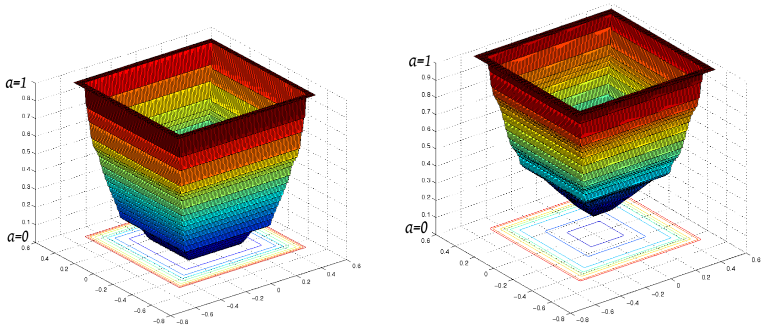


Figure: The mappings $\varepsilon \to \underline{\alpha}(V(\varepsilon))$ and $\varepsilon \to \overline{\alpha}(V(\varepsilon))$ in a 2-dimensional example.

Cloud generation

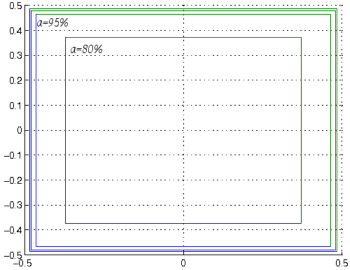


Figure: α-cuts in the plane.

Cloud generation

The expert

- initially chooses a box potential
- afterwards has the option to cut off scenarios that are not relevant
- thus specifies the uncertainty information – namely correlations – adaptively
  $\Rightarrow$ polyhedron shaped cloud

Cloud constraints

- produce the region C of worst-case relevant scenarios
- for a confidence level α one computes the solution V_α of $\overline{\alpha}(V_\alpha) = \alpha$
- define $C := \{\varepsilon | V(\varepsilon) \leq V_\alpha\}$

Design optimization

The optimization problem comes as a mixed-integer, bi-level problem, can be formulated as

$$\begin{aligned}
\min_{\theta} \ \max_{x,z,\varepsilon} \ & f(x) && \text{(objective functions)} \\
\text{s.t. } & z = Z(\theta) + D\varepsilon && \text{(table constraints)} \\
& F(x,z) = 0 && \text{(functional constraints)} \\
& V(\varepsilon) \leq V_{\alpha} && \text{(cloud constraint)} \\
& \theta \in T && \text{(selection constraints)}
\end{aligned} \tag{1}$$

Heuristics

- inner level: solved by a linear program
- outer level: 2 methods, one with SNOBFIT, one with separable underestimation

Heuristics

- SNOBFIT fits a quadratic model of the objective function and minimizes this model
- Separable underestimation takes advantage of the discrete nature of the choice variables and finds an underestimator of the objective that is easy to minimize

Heuristics

- The minimizers that result from these methods are starting points for a local search

$\Rightarrow$ hope to find the global optimum (no guarantee)

Application example

- ADCS subsystem (**A**ttitude **D**etermination and **C**ontrol **S**ystem) for the 2003 MER mission (**M**ars **E**xploration **R**over)
- The MER mission spacecraft has no main propulsion subsystem onboard, all fuel onboard the spacecraft was used only for orbit injection and for the ADCS subsystem during the cruise stage

MER spacecraft

Figure: MER spacecraft.

Underlying model

- The objective is to minimize the total mass needed for the ADCS subsystem
- The decision to make is the choice of the thrusters for the ADCS from a set of available thrusters
- Total mass consists of the total fuel and the mass of the thrusters
- The fuel is computed by a MATLAB model
  $\Rightarrow$ functional constraints for the optimization

Variable structure

- 5 fixed parameters (speed of light in vacuum, gravity constant on earth,...).
- 33 uncertain input variables (engine misalignment angles, spin rates,...).
- 3 design variables (thrust F, specific impulse I_{sp}, mass m_{thrust} of a thruster).
- 6 result variables (total mass,...).

Design table

Table: Thruster engine specifications and the linked choice variable θ

θ	**Thruster**	$F\,/\,N$	$I_{sp}\,/\,s$	$m_{thrust}\,/\,kg$
1	Aerojet MR-111C	0.27	210	0.2
2	EADS CHT 0.5	0.5	227.3	0.195
3	MBB Erno CHT 0.5	0.75	227	0.19
4	TRW MRE 0.1	0.8	216	0.5
5	Kaiser-Marquardt KMHS Model 10	1	226	0.33
6	EADS CHT 1	1.1	223	0.29
7	MBB Erno CHT 2.0	2	227	0.2
8	EADS CHT 2	2	227	0.2
9	EADS S4	4	284.9	0.29
10	Kaiser-Marquardt KMHS Model 17	4.5	230	0.38
11	MBB Erno CHT 5.0	6	228	0.22
12	EADS CHT 5	6	228	0.22
13	Kaiser-Marquardt R-53	10	295	0.41
14	MBB Erno CHT 10.0	10	230	0.24
15	EADS CHT 10	10	230	0.24
16	EADS S10 - 01	10	286	0.35
17	EADS S10 - 02	10	291.5	0.31
18	Aerojet MR-106E	12	220.9	0.476
19	SnM 15N	15	234	0.335
20	TRW MRE 4	18	217	0.5
...				

Uncertainty specification

- Given probability distributions for the single uncertain variables:
  - $U(a, b)$: uniform distribution, in (a, b),
  - $N(\mu, \sigma)$: normal distribution, with mean μ and variance σ^2,
  - $L(\mu, \sigma)$: lognormal distribution, distribution parameters μ and σ (μ and σ are the mean and standard deviation of the associated Normal-distribution),
  - $\Gamma(\alpha, \beta)$: gamma distribution, distribution parameters α and β.
- Also uncertainty information on a design variable (thrust F).

Uncertainty specification

Table: Uncertainty specifications

Variable	Probability Distribution	Unit
R	$N(1.3, 0.0013)$	m
δ_1	$N(0, 0.5)$	$\circ$
δ_2	$N(0, 0.5)$	$\circ$
ω_{spin0}	$N(12, 1.33)$	rpm
ω_{spin1}	$N(2, 0.0667)$	rpm
ω_{spin2}	$\Gamma(11, 0.25)$	rpm
ω_{spin3}	$L(2, 0.0667)$	rpm
ψ_{slew1}	$N(5, 0.5)$	$\circ$
ψ_{slew2}	$N(50.45, 5)$	$\circ$
ψ_{slew3}	$N(5.13, 0.5)$	$\circ$
...		

Optimization results

Minimize the objective function: total mass m_{tot}.

Optimal design choice: $\theta = 9$.

Variable	Nominal values	Worst case
m_{tot}	3.24	8.06

Optimization without uncertainty

Optimal design choice found for the nominal case without uncertainties: $\theta = 3$.

Variable	**Nominal values**	**Worst case**
m_{tot}	2.68	8.72

Compare it with the previous result for $\theta = 9$:

Variable	**Nominal values**	**Worst case**
m_{tot}	3.24	8.06

Conclusions

- The optimal design is sensitive to uncertainties
- Clouds can process the available uncertainty information to perform a reliable worst-case analysis linked to an adjustable confidence level
- The adaptive nature is one of the key features of the uncertainty model as it imitates real life design strategies
- The presented methods are generally applicable to problems of robust design optimization, especially with discrete design choices

http://www.mat.univie.ac.at/~mfuchs/

These slides are available on-line at:

http://www.mat.univie.ac.at/~mfuchs/